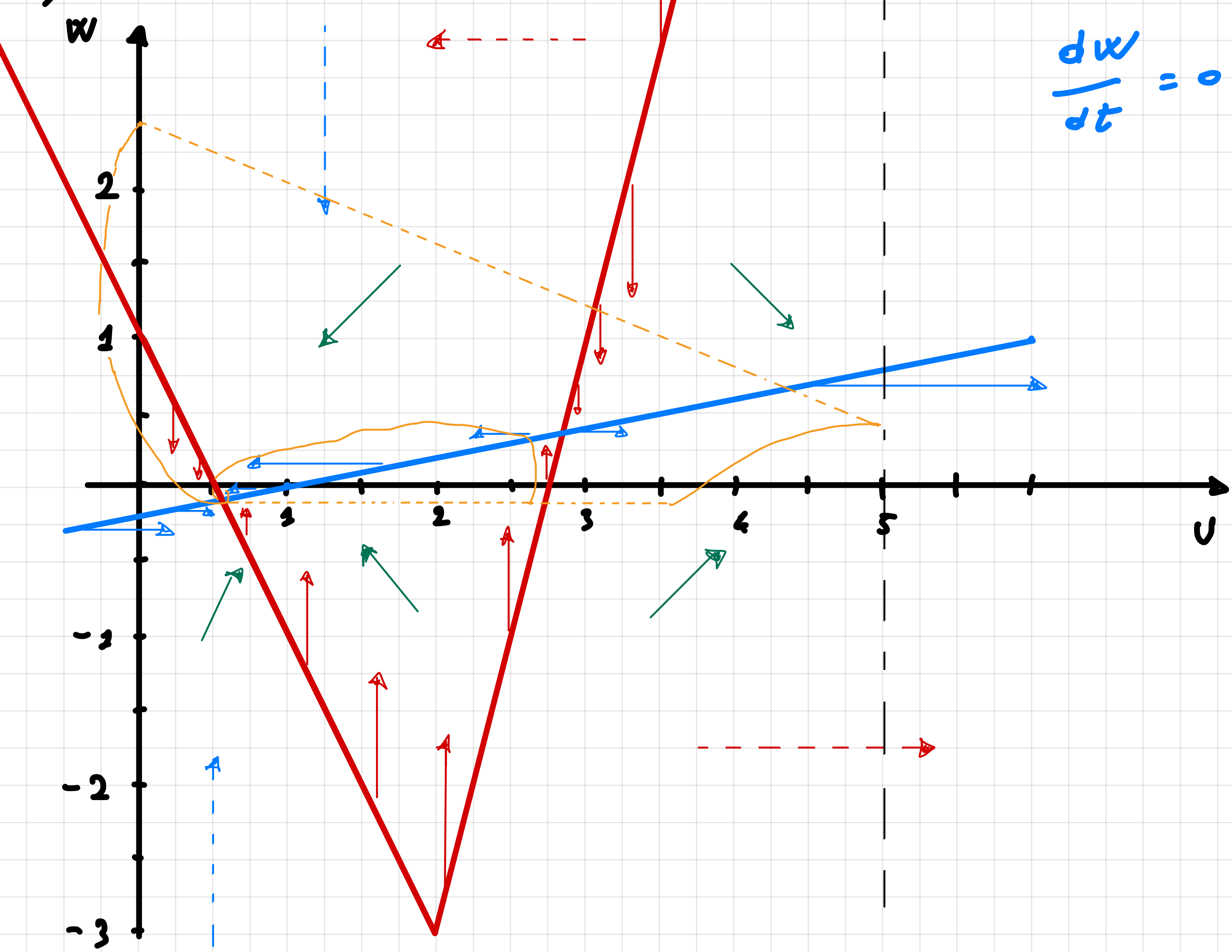
Exercise 1

a)



$\frac{du}{dt} = 0 \qquad w = F(u)$

$u \leq 2 \qquad w = 1 - 2u$

$u > 2 \qquad w = 4u - 11$

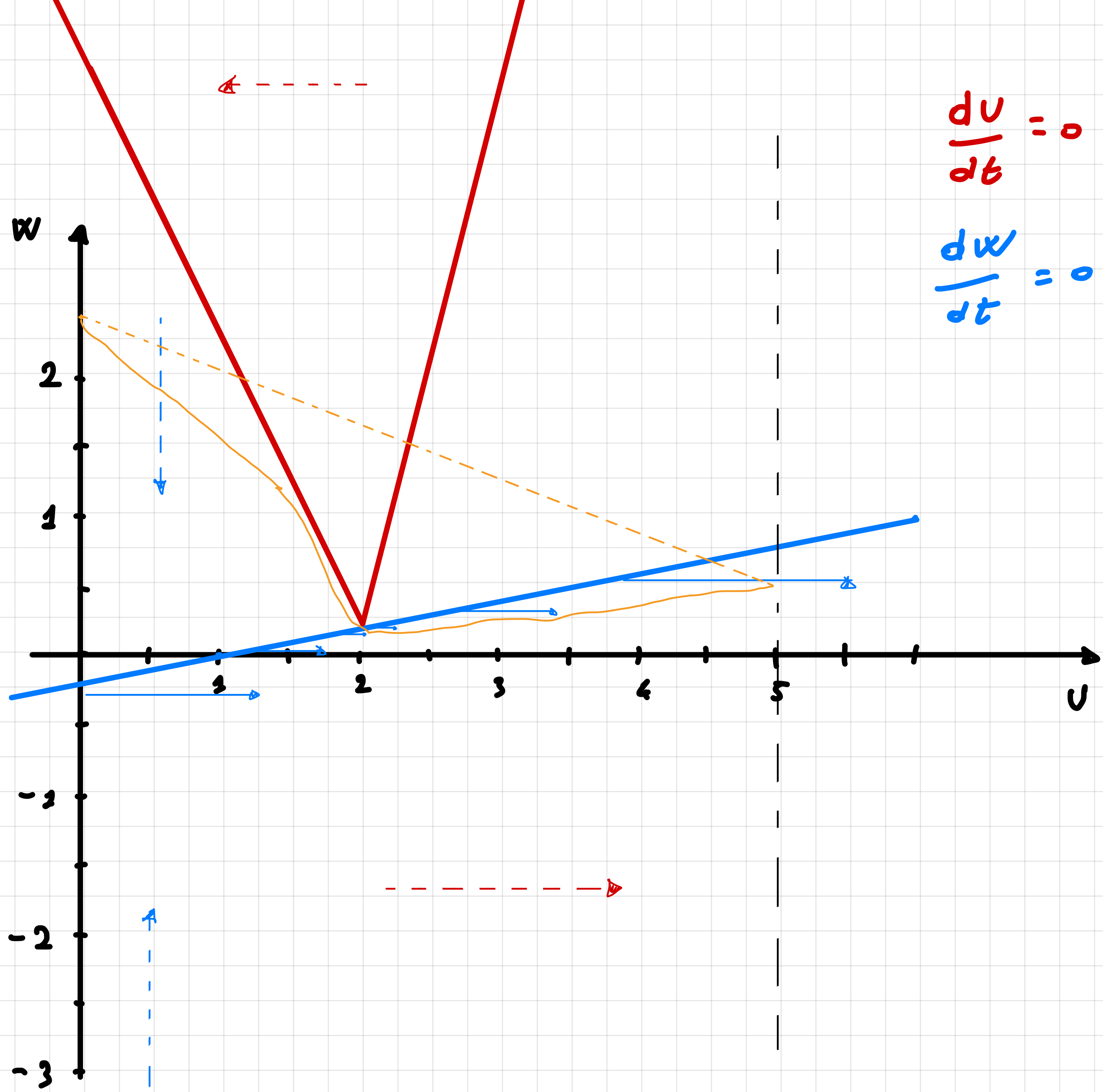
$\frac{dw}{dt} = 0$

$w = a(u-1)$

b) (See plot!)

c) (See plot!)

c)



For repeated firing, we want a configuration like the one above without a stable fixed point. For that we need

$$I = a + 3$$

8) $t \ll 1$ means w moves much faster than v.

$$t \frac{dw}{dt} = a(v-1) - w$$

w exponentially tends to $a(v-1)$ so we can approximate

$$w = a(v-1)$$

Hence

$$\frac{dv}{dt} = F(v) - a(v-1) + I$$

9) For $v < 1$:

$$-(2v-1) - a(v-1) + I = 0$$

$$-2v + 1 - v + 1 + I = 0$$

$$3v = I + 2 \qquad v = \frac{I+2}{3}$$

h)

1×1 matrix

$$\frac{d}{dx}\left(-3v+I+2\right) = -3 \quad \forall v$$

$$\det\left(-3-\lambda I\right) = 0$$

$$\lambda = -3 \qquad \tau = \frac{1}{3}$$

Around fixed point:

$$x(t) = x_0\, e^{-t/3}$$

i)

$$J = \begin{pmatrix} -2 & -1 \\ \frac{d}{\tau} & -\frac{1}{\tau} \end{pmatrix} \qquad \forall v, w$$

$$\det\left(J-\lambda I\right) = \det\begin{pmatrix} -2-\lambda & -1 \\ \frac{d}{\tau} & -\frac{1}{\tau}-\lambda \end{pmatrix} =$$

$$= (-2-\lambda)\left(-\frac{1}{t}-\lambda\right) + \frac{d}{t} =$$

$$= \frac{2}{t} + 2\lambda + \frac{\lambda}{t} + \lambda^2 + \frac{d}{t} =$$

$$= \lambda^2 + \left(2+\frac{1}{t}\right)\lambda + \left(\frac{d}{t}+\frac{2}{t}\right) =$$

$$= \lambda^2 + \left(2+\frac{1}{t}\right)\lambda + \frac{3}{t} = 0$$

Assuming $d = 1$, like before! Not specified in the exercise!

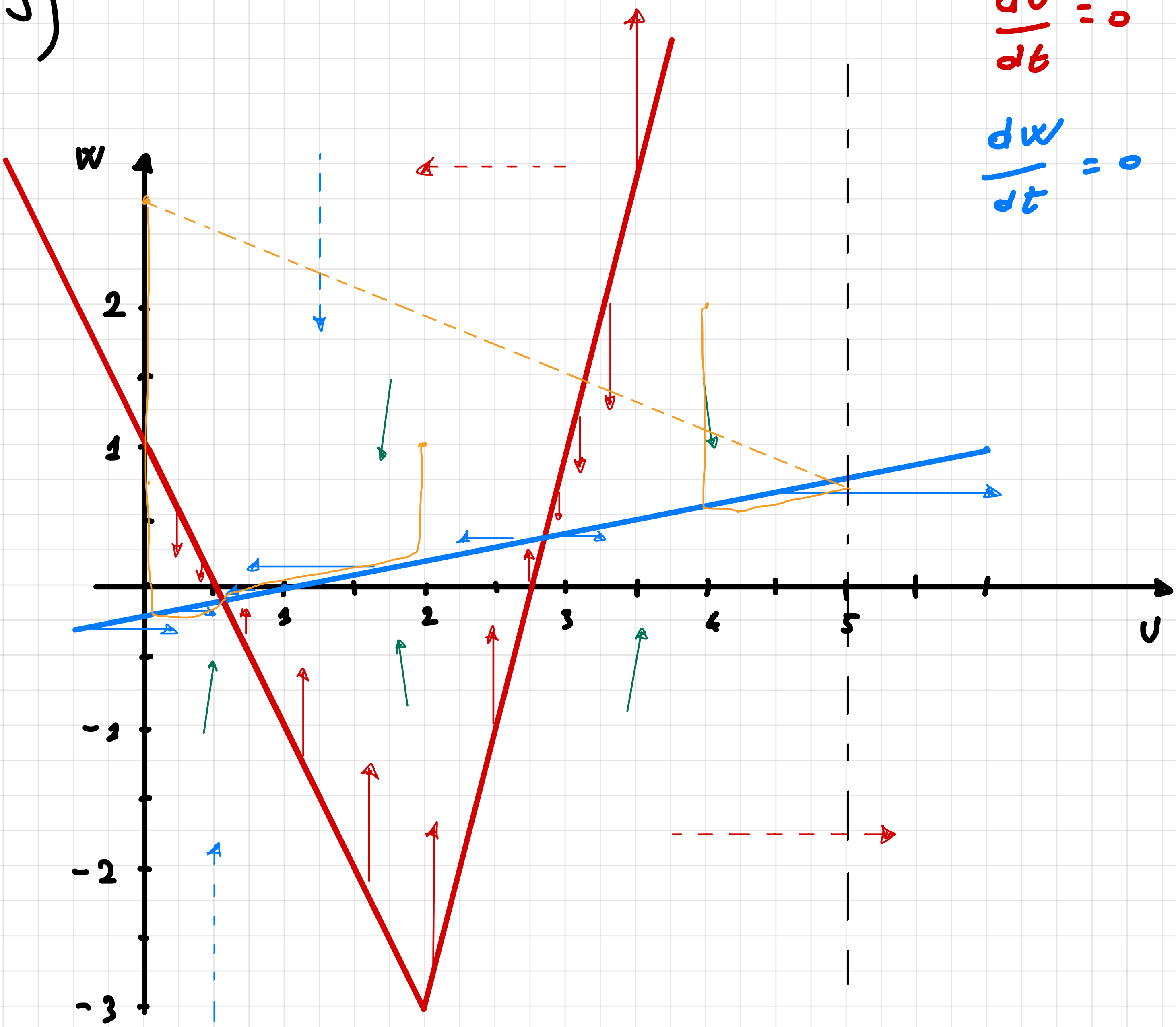
$$\lambda_\pm = \frac{-2-\frac{1}{t} \pm \sqrt{4+\frac{4}{t}+\frac{1}{t^2}-\frac{12}{t}}}{2}$$

$$\lambda_\pm = \left(-1-\frac{1}{2t}\right) \pm \frac{\sqrt{4+\frac{1}{t^2}-\frac{8}{t}}}{2}$$

$$\lambda_{\pm} = \left(-1 - \frac{1}{2t}\right) \pm \frac{\sqrt{4 + \frac{1}{t^2} - \frac{8}{t}}}{2}$$

What is the point of this...?

J)



$\frac{dU}{dt} = 0$

$\frac{dw}{dt} = 0$

Exercise 2

a)

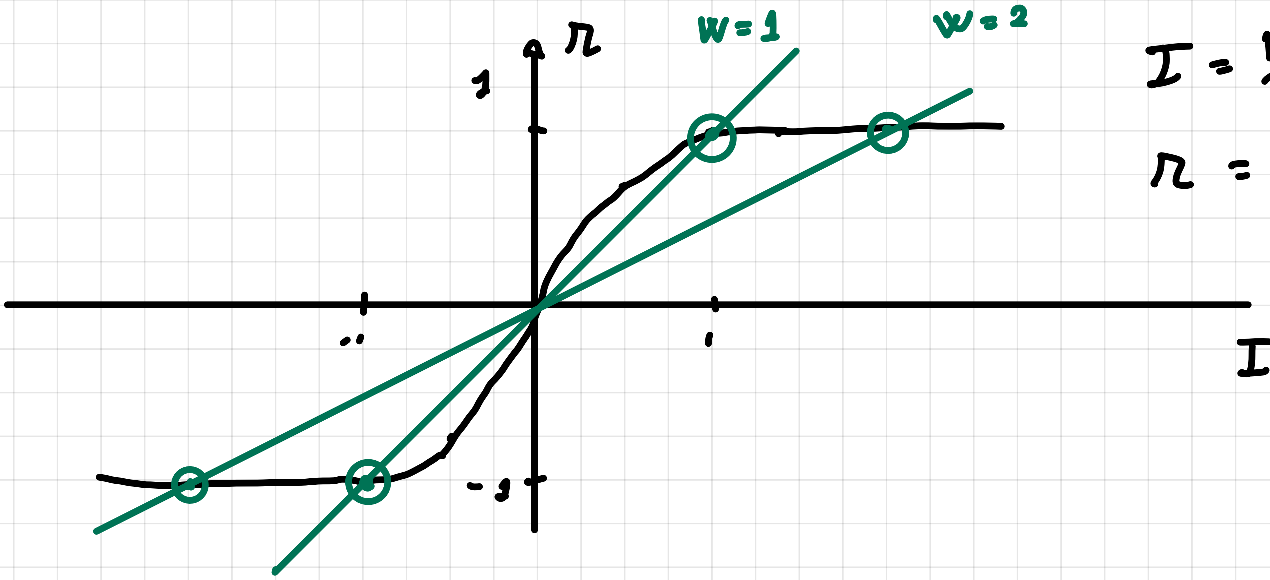
$$f'(0) = \frac{1}{2} \cdot 3 = \frac{3}{2}$$

$$f'(1) = \frac{1}{2} \cdot 3 - 3 = -\frac{3}{2}$$

$k = \frac{1}{2}$

$I = \frac{1}{2} r$

$r = 2I$



b) (see plot above!)

c)

$i \rightarrow w = \frac{1}{2}$

$I = \frac{1}{2} r$ current given rate

$r = \frac{1}{2}\left(3I - I^3\right)$ Rate given current

$$r = \frac{1}{2}\left(3 \cdot \frac{1}{2} r - \frac{1}{8} r^3\right)$$

$$\frac{3}{4} r - \frac{1}{16} r^3 = r$$

$$-\frac{1}{4} r - \frac{1}{16} r^3 = 0$$

$$\frac{1}{4} r \left(\frac{1}{4} r^2 + 1 \right) = 0$$

$$r = 0 \quad (\text{stable})$$

ii $\rightarrow$ $\quad W = \frac{3}{4} \qquad I = \frac{3}{4} r$

$$r = \frac{1}{2} \left(3 \cdot \frac{3}{4} r - \frac{27}{64} r^3 \right)$$

$$\frac{1}{8} r - \frac{27}{128} r^3 = 0$$

$$\frac{1}{8} r \left(1 - \frac{27}{16} r^2 \right) = 0$$

$r = 0$ (unstable)

$r^2 = \pm \sqrt{\frac{16}{27}}$ (stable) iii → $w = 2$

$I = 2r$

$r = \pm 1$ $\quad f(\pm 2) = \pm 1$

d) I don't know. check slides or ask online?

Exercise 3

d) We saw something similar on the exercises of the first week.

$$V_i(t) = \int_0^t w \cos(\kappa t') \sum_j S_j(t') e^{-\frac{(t-t')}{\tau}} dt'$$

For expectation we have

$$\langle S_j(t) \rangle = p^{pre}$$

hence

$$\langle V_i(t) \rangle = \int_0^t w \cos(\kappa t') N_{pre} P_{pre} e^{-\frac{(t-t')}{\tau}} dt'$$

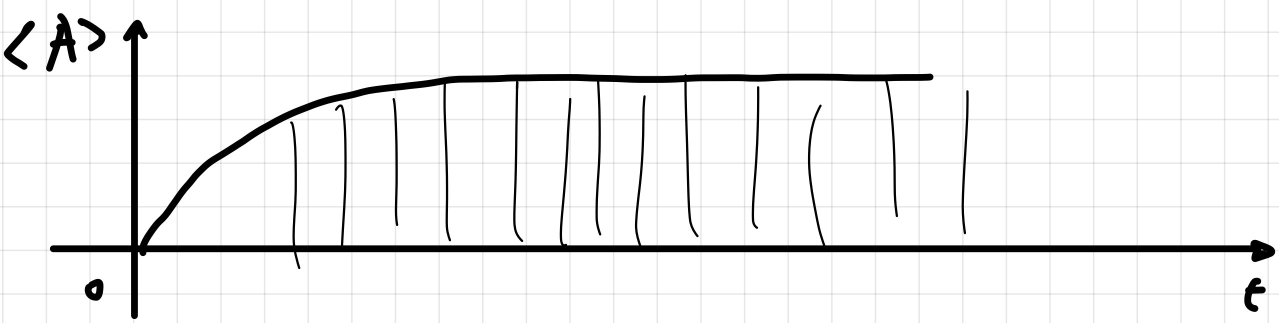
$$= N_{pre} P_{pre} w \int_0^t \cos(\kappa t') e^{-\frac{(t-t')}{\tau}} dt'$$

b) Absolutely no clue.
Woffram Alpha says there is a formula. I am BAFFLED.

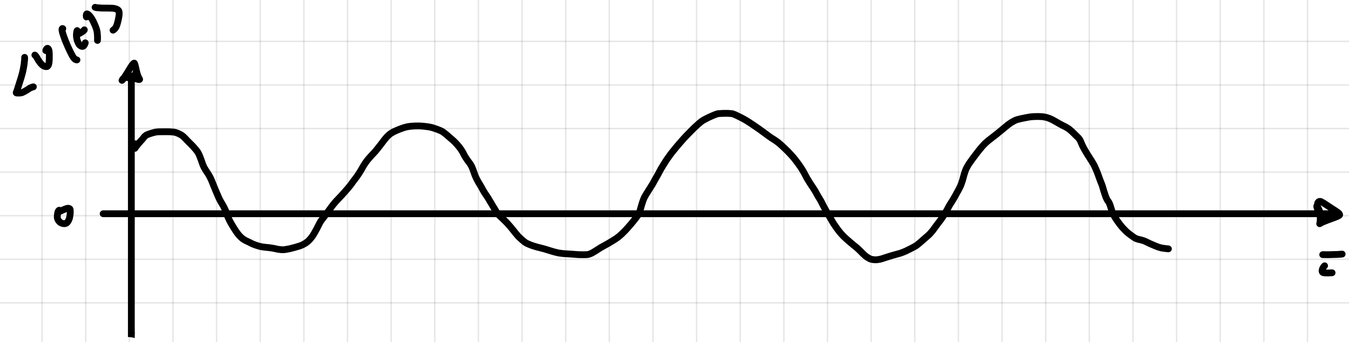
c)

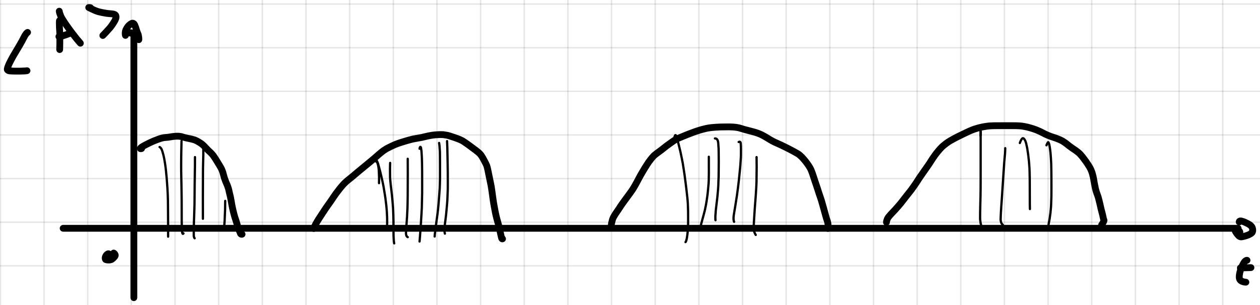
$$\langle v(t)\rangle = \frac{d_1}{1+K^2\tau^2}\left(d_2\cos(Kt) + d_3\sin(Kt) - e^{-t/\tau}\right)$$

(i) $K \ll \frac{1}{\tau}$

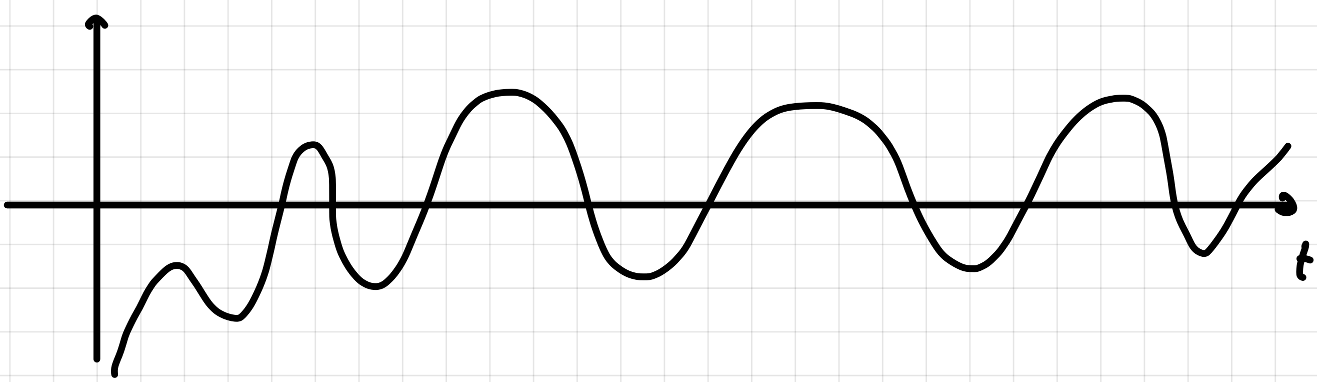


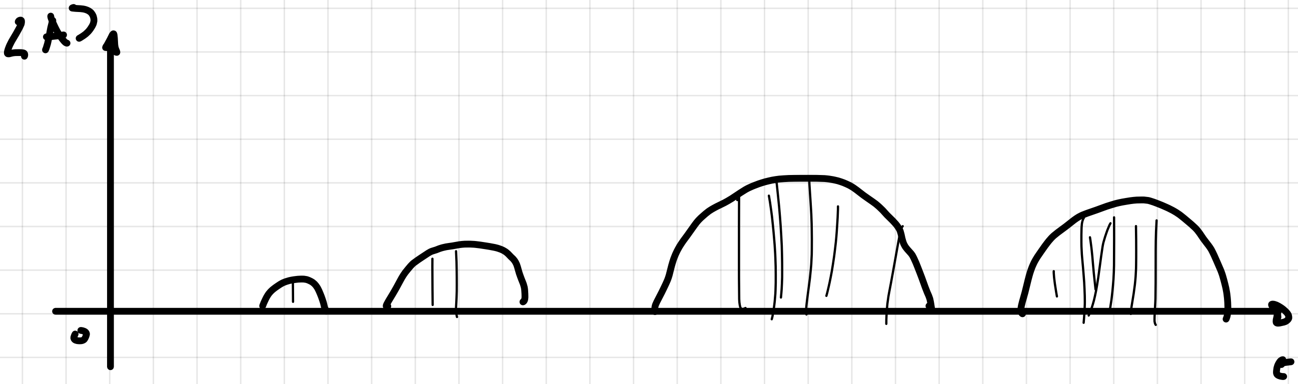
(iii) $\kappa \gg 1/\tau$





ii) $\kappa = \dfrac{1}{\tau}$





(Exam interrupted due to low morale)